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ABSTRACT

This paper documents how eight high school students in a precalculus class enhanced with graphing calculators think about functions. Three models of thinking about functions emerged from students' function images observed over a period of 9 months. In the graph model, students believed that functions are essentially graphs; in the equation model, they believed that functions are relationships between x and y expressed by equations; and in the unique correspondence model, they took "one output for every input" as the definition of a function. The vertical line test served as a link across all of the models. (SW)

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GRAPH, EQUATION AND UNIQUE CORRESPONDENCE: THREE MODELS OF STUDENTS' THINKING ABOUT FUNCTIONS IN A TECHNOLOGY-ENHANCED PRECALCULUS CLASS.

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Our area of research is aimed on developing a conceptual knowledge of functions in technology-enhanced classes. In this paper we report on the first stage of our research, documenting how students who use graphing technology think about functions. In this paper we report three models of thinking about functions (graph, equation and unique correspondence) that we found among eight high school students in a precalculus class enhanced with graphing calculators. Models emerged from their function images observed during a period of nine months.

Current efforts to reform mathematics education advocate the use of technology at all levels. In these efforts, an area of inquiry that has attracted the attention of mathematics educators is the teaching and learning of functions through technology. In general, it is expected that computers, and more recently graphing calculators are the kind of media that might help students to visualize appropriate representations of functions (Goldberg, 1987). Hence, it is conjectured that graphing capabilities of computer technology might have a positive impact on the teaching and learning of functions. These claims are supported by Dunham and Dick's review of early reports on graphing calculators (1994). Our area of research seeks to contribute to a better understanding of how students who use graphing calculators think about functions. Three models of students´ thinking about mathematical functions in a technology-enhanced precalculus class are presented here, a brief discussion of the relationships between them.

Theoretical framework

The theoretical framework developed for the research incorporated historical (cf Kleiner, 1989) and psychological contributions (processes and objects) (Sfard, 1989) to the development of functions; concept images and concept definitions (Tall, 1989); and multiple representations. We accept a constructivist view on mathematical knowledge.

The study and its methodology

Data reported in this paper belong to a larger project aimed to contribute to the teaching and learning of mathematical functions through technology. This paper involves data collected during nine months in the scholastic year 1991-1992. This initial part of the study investigated students' knowledge and development of functions in a technology-enhanced precalculus class. Students in the Calculator and Computer Precalculus Project (C^2PC, Demana & Waits, 1988) use graphing technology as an integral part of their class. Eight students from a class participating in the C^2PC were selected for case studies of their knowledge and development of functions. In particular, we investigated "*What are the concept images and the*

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concept definition of function that students in this technology-enhanced precalculus class have?" We relied on the interpretivist tradition of ethnographic research for it provides methodologies for studying the evolution of change in mathematics teaching and learning. Collection of data for each each case study involved a practice test on functions (Markovits, Eylon, and Bruckheimer, 1988) at the beginning of the study, five interviews, daily classroom observations, researcher's journal, testing materials used in class, and a student handout for extra credit.. Consideration of criteria related to the trustworthiness of the study (credibility, transferability, dependability, and confirmability) were taken into consideration as well (Lincoln & Guba, 1985).

Procedures

We discuss here only about students' protocols, since they provided the most useful information on sketching students' thinking about functions. Five protocols for interviewing students were selected or developed in the course of the study. Items were suggested by the cascading design of the study to investigate working hypothesis. Pertinent literature on functions was consulted to design the protocols (Dreyfus & Vinner, 1989; Even, 1989; Ferrini-Mundy & Graham, 1991; NCTM, 1989; Tall & Vinner, 1981). Items asked the students about the relationship between equations and functions, about the relationship between graphs and functions, to decide if some given graph was a function, to decide about the existance of a function with given algebraic features, or to provide examples of functions Items involved discrete and continuous sets and piecewise functions.

Discussion of findings

A domain analysis (Spradley, 1979) and a coding paradigm (Lincoln & Guba, 1985) was used to analyze the interviews and testing materials. Such analysis identified nine function images that students in the study associated with the concept of function (Martínez-Cruz, 1993). Resulting images were used to build a network of the concept. Links and emphases on the network (see fig. 1, 2, 3) suggested categories (graph, equation and univalence) in students' thinking. We present the categories as models of students' thinking about functions.

The models

Each model is made up of the images that emerged from all the students, however, not all images were detected on each student. Hence, these models do not state that a student can be identified as thinking about functions as one single model. On the contrary, the facts that the concept image may be incoherent, contain conflictive parts with the concept image itself or with the concept definition, or contain potential seeds for future conflict even in the learning of a formal theory (Tall & Vinner, 1981) are evident here.

Graph

The graph model refers to the graphical representation of functions. Students associated several ideas to this representation.

1) Functions can be represented by graphs.
2) Graphs can be functions (if they pass the vertical line test or the univalence criterion).
3) A graph is an intermediate step to decide whether or not an equation is a function,
4) Functions are graphs.
5) All functions can be graphed.
6) Graphs are functions.
7) Graphs come from equations.

Students' networks of functions images allowed us to identify connections and missing links among their images. One of the students, Tyler, showed a strong tendency to have a graphical representation to deal with functions (Fig. 1) (although he could talk about the equation representation or the unique correspondence criterion). This significant difference with other participants (see figures 2 and 3) is reflected on his network with a thicker line around his graph image and with an unconnected network. A second difference is how anchored his familiarity image was. He recognized a function when he has seen or graphed a similar or identical graph. Otherwise, he would reject a function based on his experience. The networks suggests also a use of the vertical line test (but not as an equivalent statement to the unique correspondence criterion).

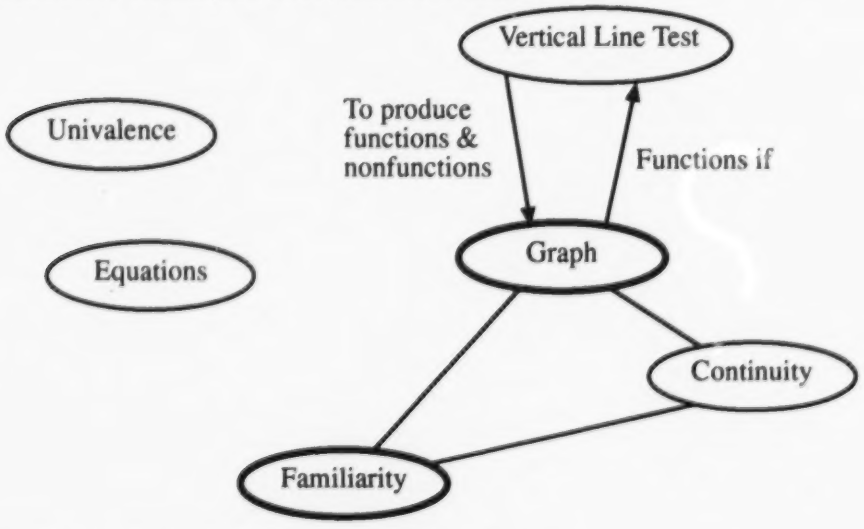

Figure 1. A student with a graph model of functions.

Equation

The equation model refers to the symbolic (algebraic) representation of functions. It appeared as a "chain" (formula) of variables and numbers. Students associated six ideas with this model.

1) A relationship between x and y.
2) Functions come from equations.
3) A means to represent functions.
4) Functions are equations.
5) Not all functions are equations.
6) Not all equations are functions.

Sara's network (Fig. 2) is a representative of an equation thinking. She relied more on an algebraic representation than on other images to deal with functions (as represented with a thicker line). A connected network is a main difference with Tyler's network and which suggests a progress on her thinking about functions. Six students showed similar networks (except for the existence of the regularity image or for their consistency on recognizing the equivalence between the unique correspondence criterion and the vertical line test). Such consistency plus a reliance on the unique correspondence criterion is a characteristic of the unique correspondence model.

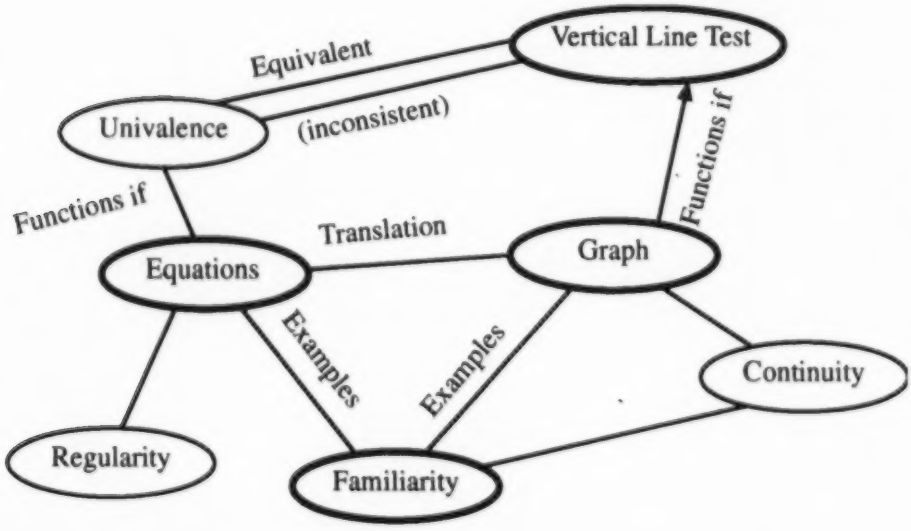

Figure 2. A student with an equation thinking of functions.

Unique correspondence

The unique correspondence model refers to the formal definition for a function introduced in this class (and at times stated as "one output for every input"). Students attached four images to this model.

1) A property of functions.
2) An implicit equivalence to the vertical line test.
3) A definition of a function.
4) A means to decide if equations or graphs (continuous or discrete) are functions.

Figure 3 shows the network of the single student who relied more strongly on the unique correspondence model than on any other model. This network also shows consistency on recognizing the unique correspondence criterion and the vertical line test (notice the thickness of both boundaries).

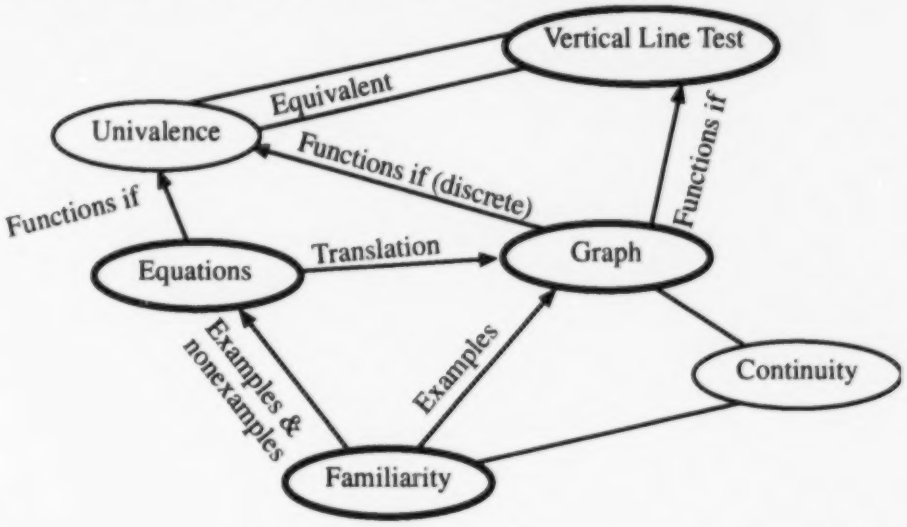

Figure 3. A student with a unique correspondence thinking of functions.

A difference between the equation and the unique correspondence models is recognizing the vertical line test and the unique correspondence as equivalent and using this recognition consistently to apply the appropriate one in a given task.

Links between the models

The vertical line test is one of the links (among others) that differentiates the networks. Students recognized the vertical line test as:

1) A means to decide whether or not a graph is a function.
2) A means to decide whether or not an equation is a function.

3) A property (condition) of functions.
4) An equivalent statement to the univalence criterion.
5) A means to produce graphs of functions or non-functions.

A second difference among the networks is the link given by translating from a given representation (algebraic usually) to another representation (graphic) to recognize functions.

Implications

Although a student may have images belonging to all three models as presented, it is noted that our data suggest that for some students one single model was more anchored in their mind than others, and they acted accordingly. Hence, they could not cope with some of the tasks presented during the interviews. Our second part of the research deals with interpreting results in the classroom. In this case, we apply the findings to teach explicitly "the knowledge and procedures of each succeeding stage of development" (Carpenter & Fennema, p. 5).

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